# FLATHEAD SOLE

By

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## SUMMARY

A model for flathead sole in the Gulf of Alaska was developed for predicting abundance and ABC levels. Analysis of maturity by age and length was completed for Gulf of Alaska flathead sole (J. Stark, pers. comm.) and included in this assessment for estimation of fishing mortality values. ABC for 2003 using $F_{40\%} = 0.417$ was estimated at 41,402 mt. Recent catches have been about 2,000 mt. Previous assessments used the 2001 survey abundance, F=0.15 (0.75*M) and knife edge selectivity at age 7 to estimate ABC at 22,684 mt (Turnock, et al. 2001). The model estimates of age 3+ biomass increased from about 254,700 mt in 1984 to about 269,700 mt in 1992, then decreased to 229,000 mt in 2002.

## INTRODUCTION

A model for flathead sole Gulf of Alaska was developed for predicting abundance and ABC levels. Flathead sole occur from Northern California to the Bering Sea, and possibly the Okhotsk Sea to Japan.

## CATCH HISTORY

Catch has been at a relatively low level, increasing to a high of about 3,100 mt in 1996, declining to 900 mt in 1999 then increasing to 2,029 mt in 2002 (through 5 October 2002). (Table 5.1). In recent years 80% to 90% of the catch has been retained.

## ABUNDANCE AND EXPLOITATION TRENDS

Survey abundance estimates were higher in 1984 and 1990 than in other years and indicate a declining trend over time(Table 5.2). Survey biomass was 170,915 mt in 2001, a decline from 207,520 mt in 1999. The 2001 survey did not cover the eastern Gulf of Alaska. The 2001 survey biomass without the estimated eastern Gulf portion was 153,747 mt. The average biomass estimated for the 1993 to 1999 surveys was used to estimate the biomass in the eastern Gulf for 2001 The eastern gulf biomass has been between 8% and 9% of the total biomass for the 1993-1999 surveys.

## ANALYTIC APPROACH

### Model Structure

The model structure was developed following Fournier and Archibald's(1982) methods, with many similarities to Methot(1990). We implemented the model using automatic differentiation software developed as a set of libraries under C++(ADModel Builder). ADModel Builder can estimate a large number of parameters in a non-linear model using automatic differentiation software extended from Greiwank and Corliss(1991) and developed into C++ class libraries. This software provides the derivative calculations needed for finding the objective function via a quasi-Newton function minimization routine(e.g., Press et al. 1992). The model implementation language (ADModel Builder) gives simple and rapid access to these routines and provides the ability to estimate the variance-covariance matrix for all parameters of interest. Details of the population dynamics and estimation equations, description of variables and likelihood equations are presented in Appendix A (Tables A.1, A.2 and A.3) . There were a total of 66 parameters estimated in the model (Table A.4). The instantaneous natural mortality rate, catchability for the survey and the von Bertalanffy growth parameters were fixed in the model (Table A.5).

### Model assumptions

All emphasis values were 1 except for the catch likelihood where a high emphasis was used to fit the catch closely, while allowing some deviation from the observed catch. Recruitments for the last three years in the model were constrained to be close the mean log recruitment, since there is little information for their estimation. The model estimates size compositions using a fixed length-age transition matrix estimated from the 1984, 1993 and 1996 survey length and age data combined. The distribution of lengths within ages was assumed to be normal with cv's estimated from the length at age data of 0.10 for younger ages and 0.08 for older ages.

## Data Sources

The model simulates the dynamics of the population and compares the expected values of the population characteristics to those observed from surveys and fishery sampling programs.

The following data sources were used in the model:

| Data component | Years |
|---|---|
| Fishery catch (Table 5.1) | 1978-2002 |
| NMFS triennial trawl survey biomass and S.E. (Table 5.2) | 1984,1987,1990,1993,1996,1999,2001 |
| Fishery size compositions | 1985-2002 |
| NMFS triennial trawl survey size compositions | 1987,1990,1999,2001 |
| NMFS triennial trawl survey age composition data | 1984,1993,1996. |

## Natural mortality, Age of recruitment, and Maximum Age

Natural mortality was fixed at 0.2. Age at recruitment was set at three in the model due to the small number of fish caught at younger ages. The maximum age of flathead sole based on otolith age determinations was 25 years.

## Weight at Age

The weight-length relationship for flathead sole is, $W = 0.00428\ L^{3.2298}$, for both sexes combined where weight is in grams and length in centimeters.

## Selectivity

The selectivity curve for the fishery and the survey was estimated using a two parameter ascending logistic function estimated separately for males and females (Figure 5.1).

## Growth

The growth parameter $L_{inf}$ was estimated at 44.37 cm for females and 37.36 cm for males (Figure 5.2). The length at age 2 was estimated at 10.17 cm for males and 13.25 cm for females. The growth parameter k was estimated at 0.157 for females and 0.204 for males. Length at time t was modeled as:

$$L_t = L_{\max} + (L_1 - L_{\max}) * \exp(-k(t-1)) .$$

## Maturity

Gulf of Alaska flathead sole maturity was estimated using histological analysis of ovaries collected in a January 1999 study (J. Stark, pers. comm.). A total of 180 samples were analyzed for estimation of age at maturity. Size at 50% mature was estimated to be 32.0 cm with a slope of

0.775 from a sample of 208 fish. Age at 50% mature was 7.9 with a slope of 1.115. Size at 50% mature was also estimated at 32.0 cm for Bering sea flathead sole, however, age at 50% mature was 9.7 due to slower growth in the Bering sea.

## RESULTS

Fits to the size composition data from the fishery are shown in Figure 5.3 for females and Figure 5.4 for males. The fit to the survey size composition data are in Figure 5.5 for females and Figure 5.6 for males. The survey age composition data are shown in Figures 5.7 and 5.8. Data in the late 1980's indicate one or more large cohorts. However, later length and age data do not contain the larger, older fish that would result from those large recruitments. The model estimates lower recruitments in the late 1980's to fit the later length and age data better.

### Model estimates of biomass

The model estimates of age 3+ biomass increased from about 254,700 mt in 1984 to about 269,700 mt in 1992, then decreased to 229,000 mt in 2002 (Table 5.3 and Figure 5.9). The fit to the survey biomass estimates is shown in Figure 5.10. The model estimates an increasing biomass despite the high observed biomass in 1984 and lower biomass in 1987 due to the survey length and age data, which indicate a relatively large cohort moving through the population during that time period.

### Model estimates of recruitment

The model estimates of age 3 recruits were lower than average for the last seven years from 1996 to 2002 (Table 5.3 and Figure 5.11). Recruitments in 2000 to 2002 were constrained to be close to the historical harmonic mean recruitment. This was done as a precautionary approach since the harmonic mean recruitment is less than the arithmetic mean recruitment.

### Spawner-Recruit Relationship

No spawner-recruit curve was used in the Model. Recruitments were estimated as deviations from a mean value on a log scale.

## REFERENCE FISHING MORTALITY RATES AND YIELDS

Reliable estimates of biomass, $B_{35\%}$, $F_{35\%}$ and $F_{40\%}$, are available, and current biomass is greater than $B_{40\%}$. Therefore, flathead sole is in tier 3a of the ABC and overfishing definitions. Under this definition, $F_{ofl}$= $F_{35\%}$, and $F_{ABC}$ is less than or equal to $F_{40\%}$.

Yield for 2003 using $F_{40\%}$ = 0.417 was estimated at 41,402 mt. Yield at $F_{35\%}$ = 0.546 was estimated at 51,556 mt. ABC was 22,700 mt using only the 2001 survey and F=0.15 (Turnock, et al., 2001). Fishing mortality values are higher than previously estimated because the age at 50% selected in the fishery is about 9.5 years, while the age at 50% mature is about 8 years. The fishery selectivities reach 95% at about age 13 for females. The F at age 10 is about 0.246.

## MAXIMUM SUSTAINABLE YIELD

Since there is no estimate of the spawner-recruit relationship for flathead sole, no attempt has been made to estimate MSY. However, using the projection model described in the next section, spawning biomass with F=0 was estimated at 95,406 mt. $B_{35\%}$ (equilibrium female spawning biomass with fishing at $F_{35\%}$) is estimated at 33,392 mt.

## PROJECTED CATCH AND ABUNDANCE

A standard set of projections is required for each stock managed under Tiers 1, 2, or 3 of Amendment 56. This set of projections encompasses seven harvest scenarios designed to satisfy the requirements of Amendment 56, the National Environmental Protection Act, and the Magnuson-Stevens Fishery Conservation and Management Act (MSFCMA).

For each scenario, the projections begin with the vector of 2002 numbers at age estimated in the assessment. This vector is then projected forward to the beginning of 2003 using the schedules of natural mortality and selectivity described in the assessment and the best available estimate of total (year-end) catch for 2002. In each subsequent year, the fishing mortality rate is prescribed on the basis of the spawning biomass in that year and the respective harvest scenario. In each year, recruitment is drawn from an inverse Gaussian distribution whose parameters consist of maximum likelihood estimates determined from recruitments estimated in the assessment. Spawning biomass is computed in each year based on the time of peak spawning and the maturity and weight schedules described in the assessment. Total catch is assumed to equal the catch associated with the respective harvest scenario in all years. This projection scheme is run 1000 times to obtain distributions of possible future stock sizes, fishing mortality rates, and catches.

Five of the seven standard scenarios will be used in an Environmental Assessment prepared in conjunction with the final SAFE. These five scenarios, which are designed to provide a range of harvest alternatives that are likely to bracket the final TAC for 2003, are as follow ("*max* $F_{ABC}$" refers to the maximum permissible value of $F_{ABC}$ under Amendment 56):

> *Scenario 1*: In all future years, $F$ is set equal to *max* $F_{ABC}$. (Rationale: Historically, TAC has been constrained by ABC, so this scenario provides a likely upper limit on future TACs.)
>
> *Scenario 2*: In all future years, $F$ is set equal to a constant fraction of *max* $F_{ABC}$, where this fraction is equal to the ratio of the $F_{ABC}$ value for 2003 recommended in the assessment to the *max* $F_{ABC}$ for 2003. (Rationale: When $F_{ABC}$ is set at a value below *max* $F_{ABC}$, it is often set at the value recommended in the stock assessment.)
>
> *Scenario 3*: In all future years, $F$ is set equal to 50% of max $F_{ABC}$. (Rationale: This scenario provides a likely lower bound on $F_{ABC}$ that still allows future harvest rates to be adjusted downward when stocks fall below reference levels.)
>
> *Scenario 4*: In all future years, $F$ is set equal to the 1995-1999 average $F$. (Rationale: For some stocks, TAC can be well below ABC, and recent average $F$ may provide a better indicator of $F_{TAC}$ than $F_{ABC}$.)
>
> *Scenario 5*: In all future years, $F$ is set equal to zero. (Rationale: In extreme cases, TAC may be set at a level close to zero.)

Two other scenarios are needed to satisfy the MSFCMA's requirement to determine whether a stock is currently in an overfished condition or is approaching an overfished condition. These two scenarios are as follow (for Tier 3 stocks, the MSY level is defined as $B_{35\%}$):

*Scenario 6*: In all future years, $F$ is set equal to $F_{OFL}$. (Rationale: This scenario determines whether a stock is overfished. If the stock is expected to be above ½ of its MSY level in 2003 and above its MSY level in 2013 under this scenario, then the stock is not overfished.)

*Scenario 7*: In 2003 and 2004, $F$ is set equal to *max* $F_{ABC}$, and in all subsequent years, $F$ is set equal to $F_{OFL}$. (Rationale: This scenario determines whether a stock is approaching an overfished condition. If the stock is expected to be above its MSY level in 2015 under this scenario, then the stock is not approaching an overfished condition.)

Projected catch and abundance were estimated using $F_{40\%}$, F equal to the average F from 1995 to 1999, F equal to one half $F_{40\%}$, and F=0 from 2003 to 2007 (Table 4.10). Under scenario 6 above, the year 2003 female spawning biomass is 95,406 mt and the year 2013 spawning biomass is 35,005 mt, above the $B_{35\%}$ level of 33,392 mt. For scenario 7 above, the year 2015 spawning biomass is 34,949 mt also above $B_{35\%}$.

## ACCEPTABLE BIOLOGICAL CATCH

ABC for 2003 using $F_{40\%}$ = 0.417 was estimated at 41,402 mt. In last year's assessment ABC for 2002 using $F_{40\%}$ = 0.15 was estimated at 22,684 mt (Turnock, et al. 2001).
The ABC by management area using $F_{40\%}$ was estimated by calculating the fraction of the 2001 survey biomass in each area and applying that fraction to the ABC:

Flathead sole ABC (mt) by INPFC area

| | Western | Central | West Yakutat | East Yakutat/SE | Total |
|---|---|---|---|---|---|
| ABC 2003 | 16,421 | 20,823 | 2,904 | 1,254 | 41,402 |

## OVERFISHING DEFINITION

Yield at $F_{35\%}$ = 0.546 was estimated at 51,556 mt.

## SUMMARY

Table 4.10 shows a summary of model results.

## Literature cited

Fournier, D.A. and C.P. Archibald. 1982. A general theory for analyzing catch-at-age data. Can. J.Fish.Aquat.Sci. 39:1195-1207.

Greiwank, A. and G.F. Corliss(eds). 1991. Automatic differentiation of algorithms: theory, implementation and application. Proceedings of the SIAM Workshop on the Automatic Differentiation of Algorithms, held Jan. 6-8, Breckenridge, CO. Soc. Indust. And Applied Mathematics, Philadelphia.

Methot, R. D. 1990. Synthesis model: An adaptable framework for analysis of diverse stock assessment data. Int. N. Pac. Fish. Comm. Bull. 50:259-277.

Press, W.H., S.A. Teukolsky, W.T.Vetterling, B.P. Flannery. 1992. Numerical Recipes in C. Second Ed. Cambridge Univ. Press. 994 p.

Table 5.1. Catch of flathead sole in the Gulf of Alaska from 1978 to 5 October 2002.

| Year | Catch(mt) |
|---|---|
| 1978 | 451 |
| 1979 | 165 |
| 1980 | 2,068 |
| 1981 | 1,070 |
| 1982 | 1,368 |
| 1983 | 1,079 |
| 1984 | 548 |
| 1985 | 319 |
| 1986 | 147 |
| 1987 | 150 |
| 1988 | 520 |
| 1989 | 747 |
| 1990 | 1,447 |
| 1991 | 1,717 |
| 1992 | 2,034 |
| 1993 | 2,366 |
| 1994 | 2,580 |
| 1995 | 2,181 |
| 1996 | 3,107 |
| 1997 | 2,446 |
| 1998 | 1,742 |
| 1999 | 900 |
| 2000 | 1,547 |
| 2001 | 1,911 |
| 2002 | 2,029 |

Table 5.2. Biomass estimates and standard errors from NMFS bottom trawl surveys. Note that in 2001 the eastern GOA was not surveyed. The estimated eastern survey biomass based on previous surveys was added to the 2001 biomass (153,747 mt), which increased the total by about 11%.

| Survey year | Biomass(mt) | Standard deviation |
|---|---|---|
| 1984 | 249,335 | 30,421 |
| 1987 | 179,820 | 19,227 |
| 1990 | 243,067 | 28,879 |
| 1993 | 188,592 | 24,458 |
| 1996 | 205,485 | 18,429 |
| 1999 | 207,520 | 24,418 |
| 2001 | 170,915 | 20,510 |

Table 5.3. Estimated age 3+ population biomass(mt), female spawning biomass(mt) and age 3 recruits(1,000's).

| Year | age 3+ biomass | Female spawning biomass | Age 3 recruits (1,000's) |
|---|---|---|---|
| 1984 | 254,735 | 79,566 | 129,440 |
| 1985 | 261,873 | 93,514 | 221,171 |
| 1986 | 266,690 | 103,050 | 227,903 |
| 1987 | 267,000 | 107,993 | 146,831 |
| 1988 | 268,183 | 109,463 | 244,556 |
| 1989 | 269,130 | 108,745 | 242,984 |
| 1990 | 269,434 | 107,470 | 217,173 |
| 1991 | 268,843 | 106,035 | 211,606 |
| 1992 | 269,708 | 104,735 | 257,099 |
| 1993 | 266,693 | 103,842 | 136,426 |
| 1994 | 264,330 | 103,492 | 213,182 |
| 1995 | 264,915 | 103,375 | 290,107 |
| 1996 | 263,266 | 103,398 | 164,928 |
| 1997 | 257,772 | 102,428 | 120,419 |
| 1998 | 252,138 | 101,367 | 156,521 |
| 1999 | 246,331 | 100,737 | 157,713 |
| 2000 | 241,122 | 100,811 | 163,614 |
| 2001 | 235,156 | 99,634 | 159,822 |
| 2002 | 229,070 | 96,928 | 156,038 |

Table 5.4. Projected female spawning biomass and yield from 2003 to 2007.

| Year | Female spawning biomass(mt) | Yield(mt) |
|---|---|---|
| F=F40% | | |
| 2003 | 93,524 | 41,402 |
| 2004 | 68,099 | 29,400 |
| 2005 | 52,892 | 21,910 |
| 2006 | 44,128 | 17,481 |
| 2007 | 39,392 | 14,985 |
| | | |
| F=0.017(avg F) | | |
| 2003 | 93,524 | 2,013 |
| 2004 | 90,312 | 1,948 |
| 2005 | 87,709 | 1,881 |
| 2006 | 85,623 | 1,823 |
| 2007 | 84,125 | 1,776 |
| | | |
| F=0.5 F40% | | |
| 2003 | 93,524 | 22,465 |
| 2004 | 78,720 | 18,693 |
| 2005 | 68,106 | 15,813 |
| 2006 | 60,717 | 13,740 |
| 2007 | 55,844 | 12,319 |
| | | |
| F=0 | | |
| 2003 | 93,524 | 0 |
| 2004 | 91,458 | 0 |
| 2005 | 89,822 | 0 |
| 2006 | 88,530 | 0 |
| 2007 | 87,676 | 0 |

Table 5.6. Summary of results of flathead sole assessment in the Gulf of Alaska.

| | | |
|---|---|---|
| Natural Mortality | | 0.2 females and males |
| Age of full(95%) fishery selection | | 13 females, 14 males |
| Reference fishing mortalities | | |
| | $F_{40\%}$ | 0.417 |
| | $F_{35\%}$ | 0.546 |
| Biomass at MSY | | N/A |
| Equilibrium unfished Spawning biomass | | 95,406 mt |
| $B_{35\%}$ Spawning biomass fishing at $F_{35\%}$ | | 33,392 mt |
| $B_{40\%}$ Spawning biomass fishing at $F_{40\%}$ | | 38,163 mt |
| Projected 2003 biomass | | |
| | Total(age 3+) | 224,570 mt |
| | Spawning | 93,524 mt |
| | Exploitable | 132,264 mt |
| Overfishing level for 2003 | | 51,556 mt |

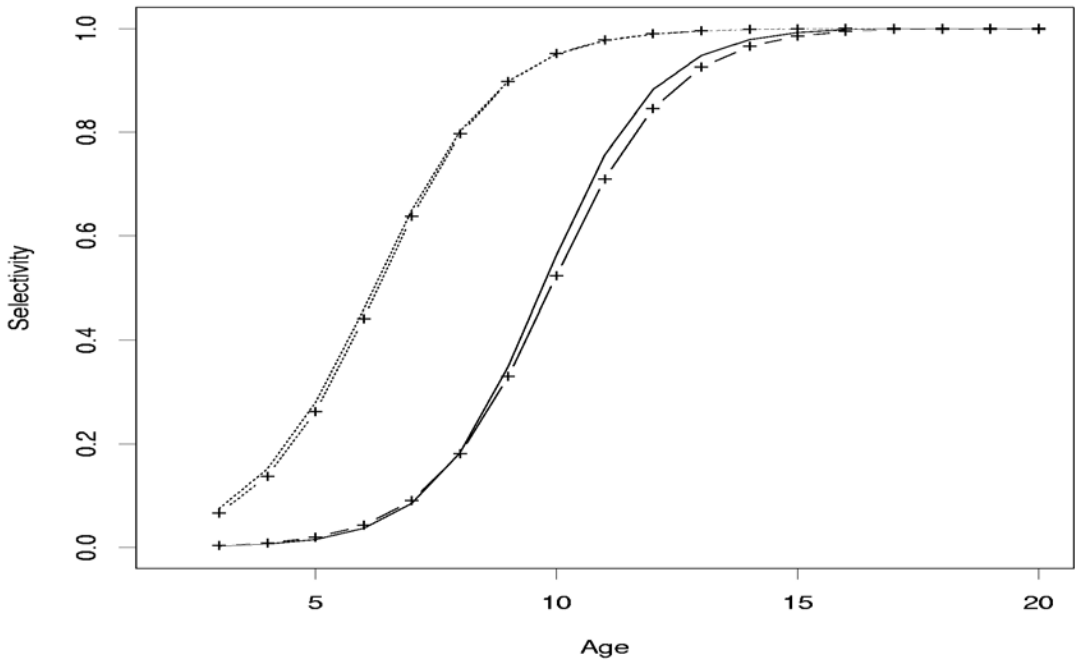


Figure 5.1. Selectivities for the survey (dotted line) and fishery (solid line). Male curve with +.

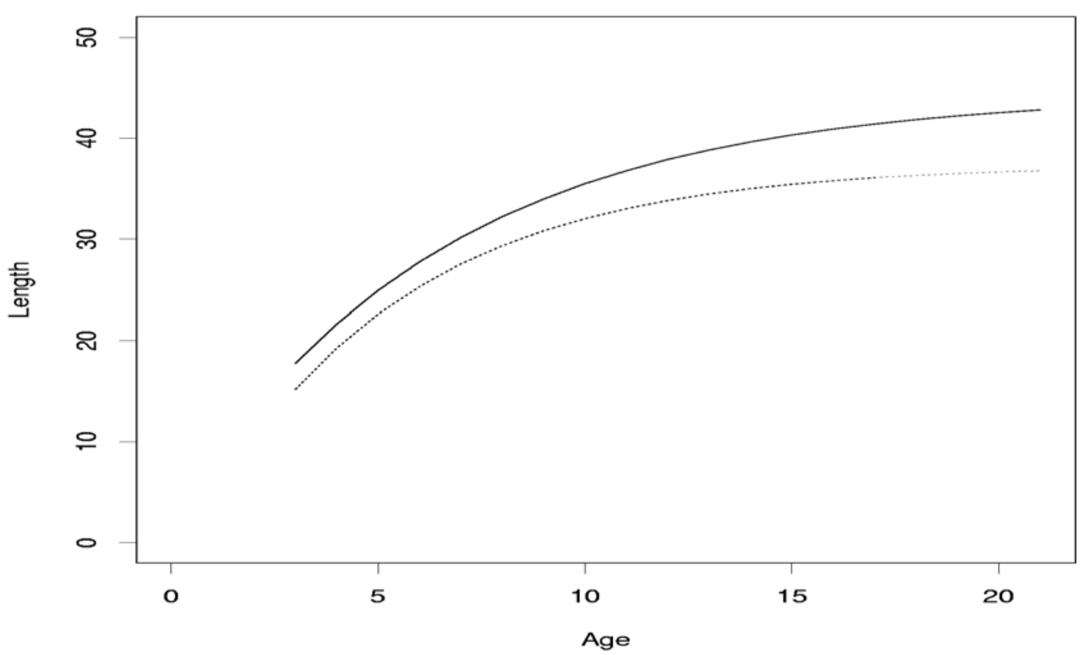


Figure 5.2. Growth for flathead sole, females solid line, males dotted line.

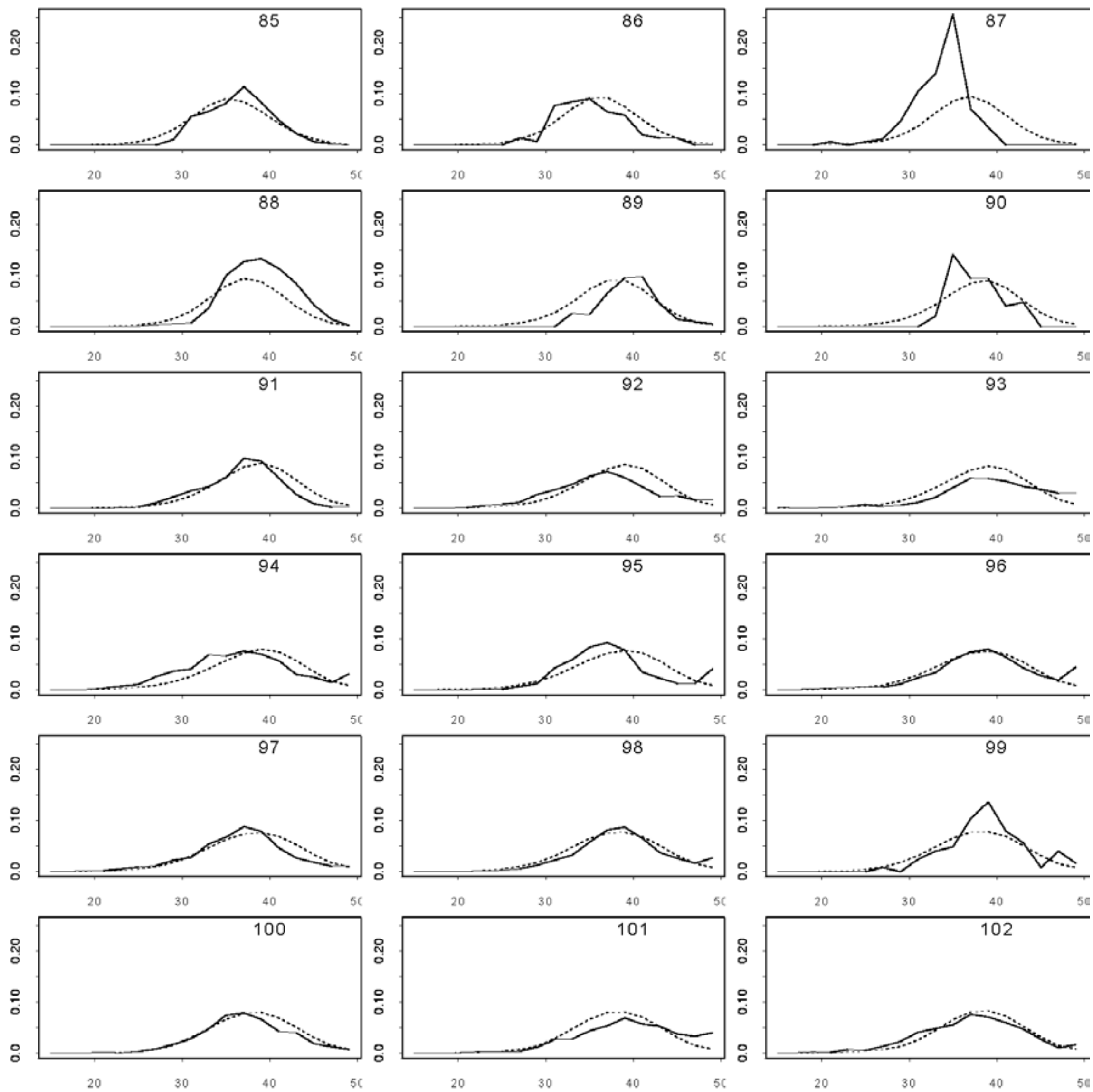


Figure 5.3. Fit to female fishery length composition data.

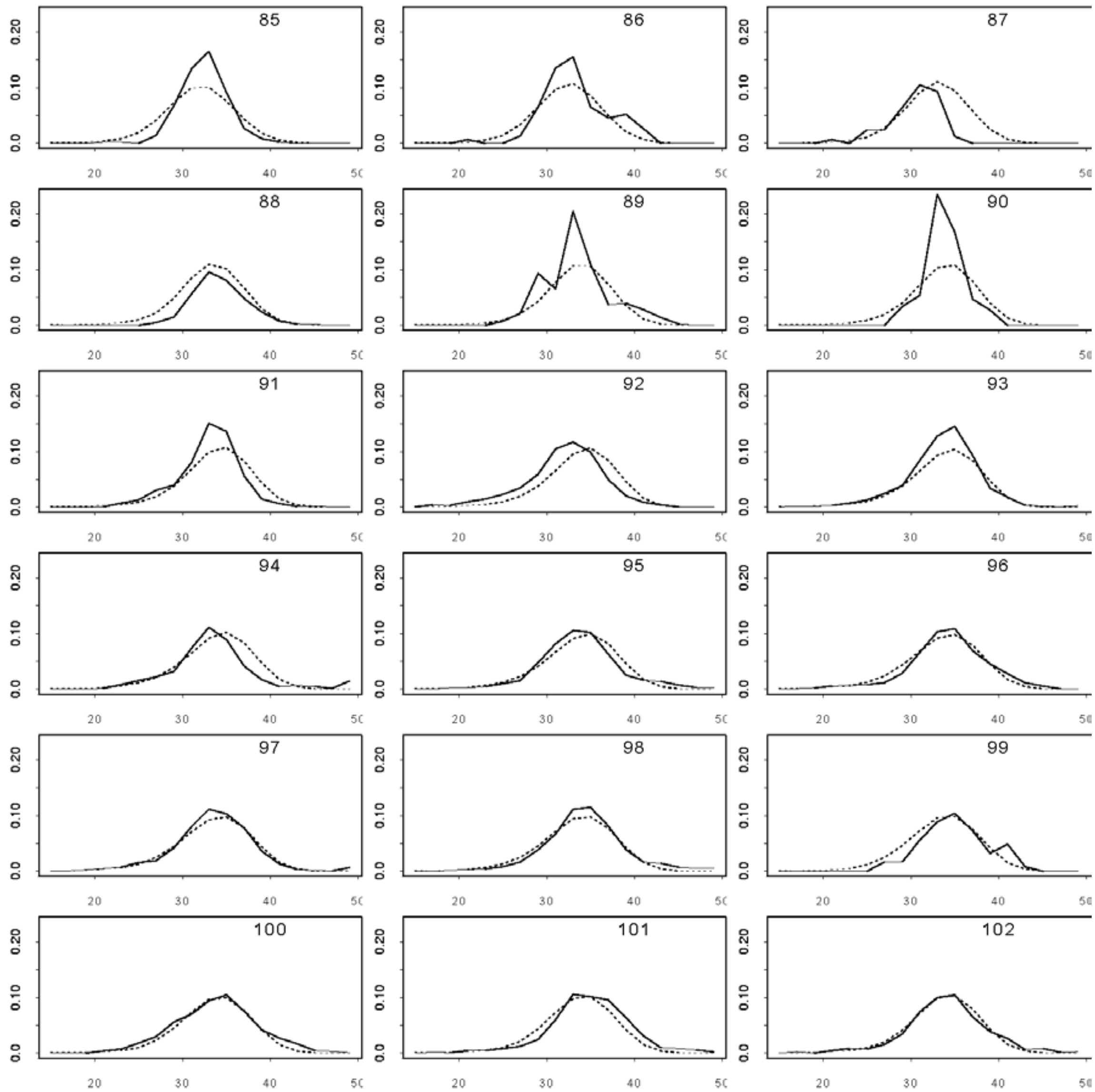


Figure 5.4. Fit to male fishery length composition data.

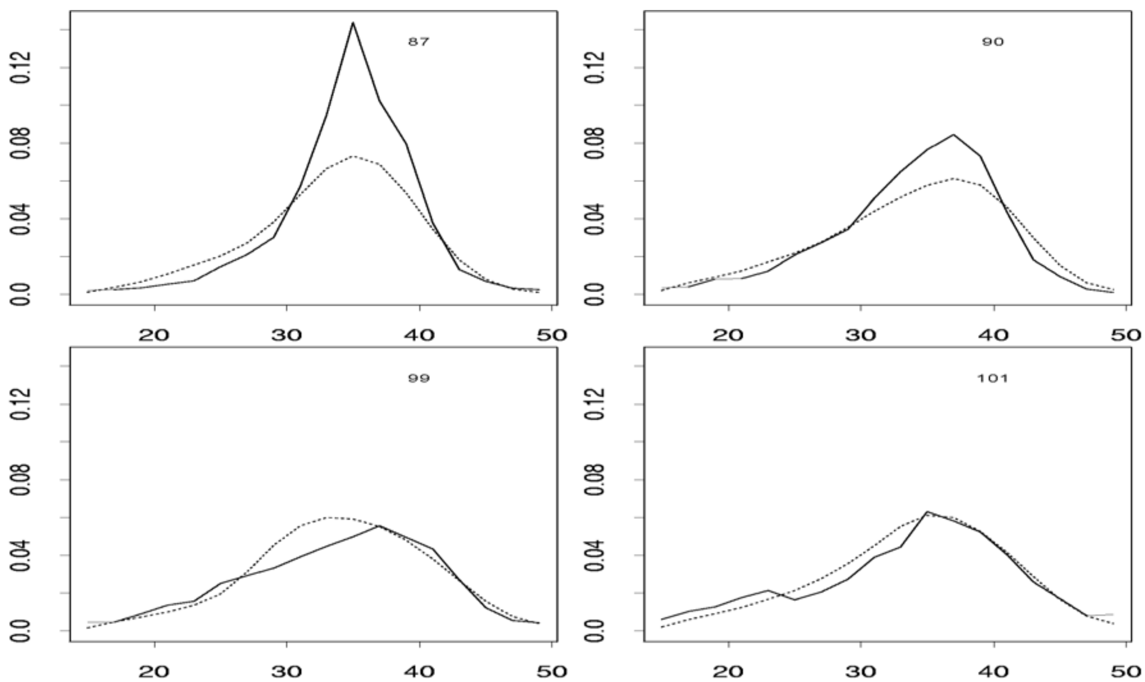


Figure 5.5. Fit to the female survey length composition data.

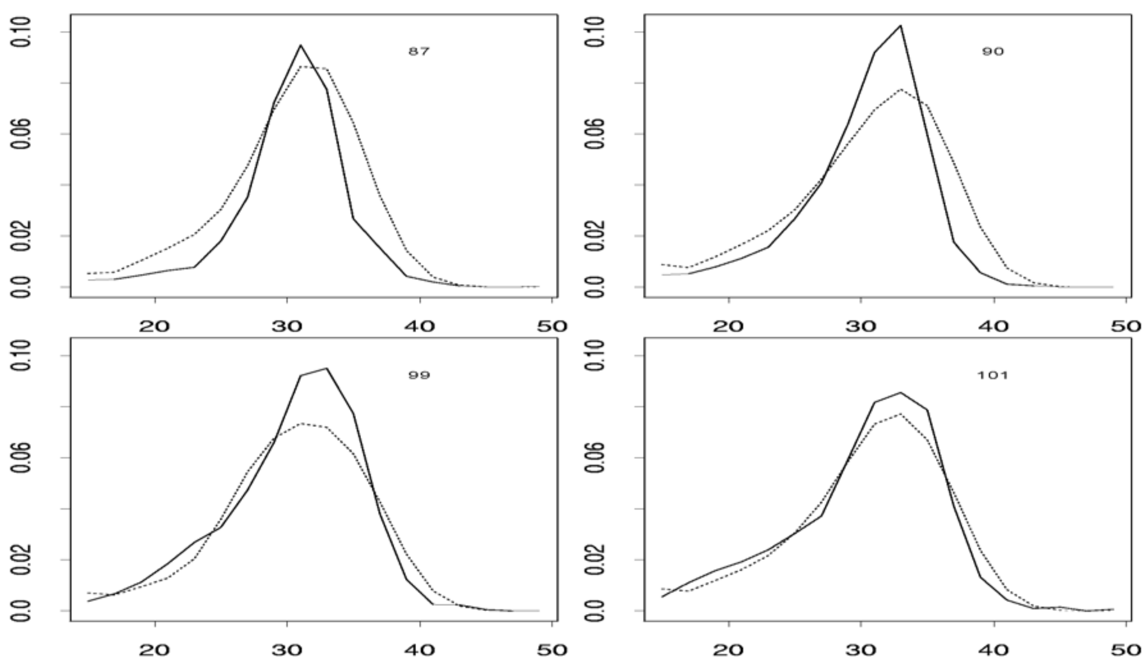


Figure 5.6. Fit to the male survey length composition data.

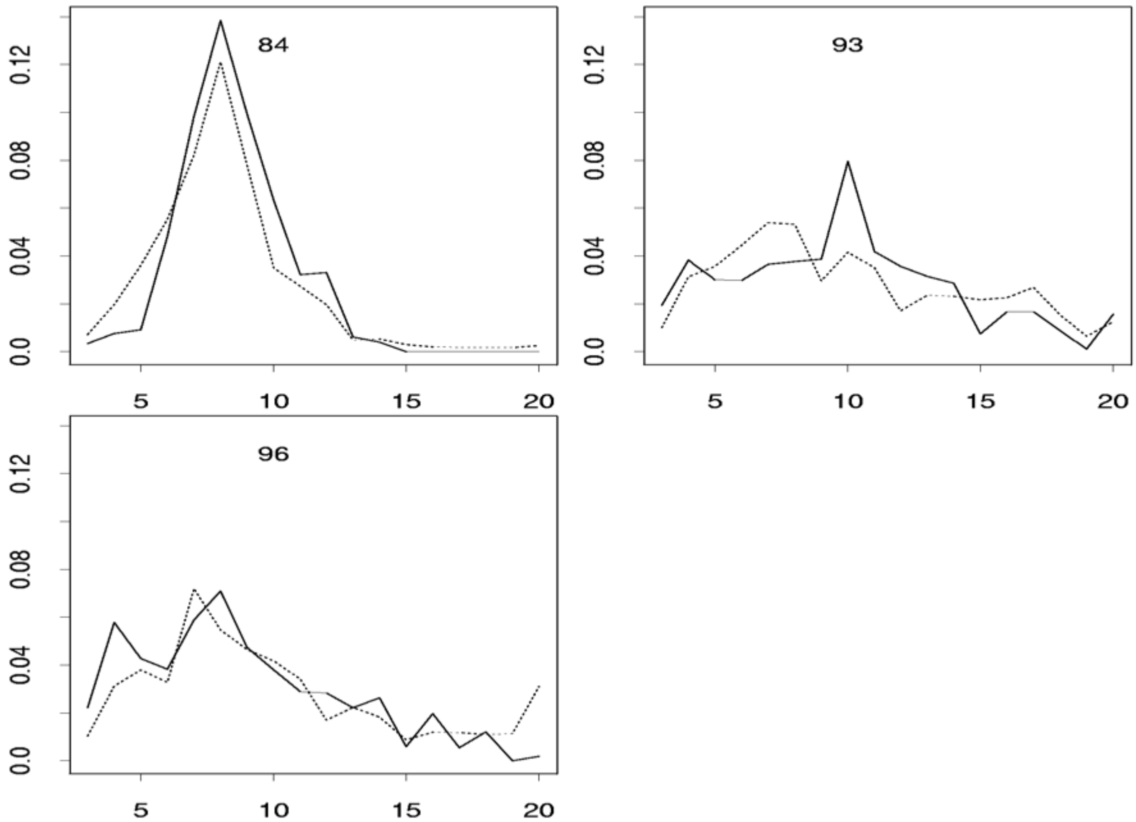


Figure 5.7. Fit to the female survey age composition data.

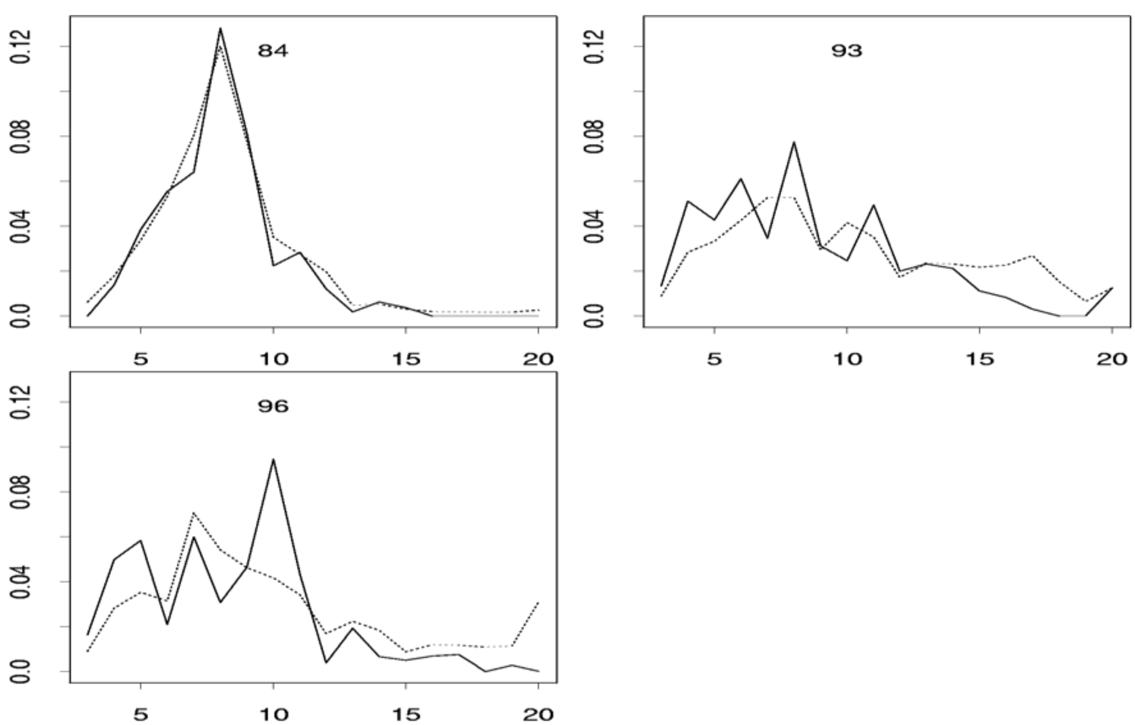


Figure 5.8. Fit to the male survey age composition data.

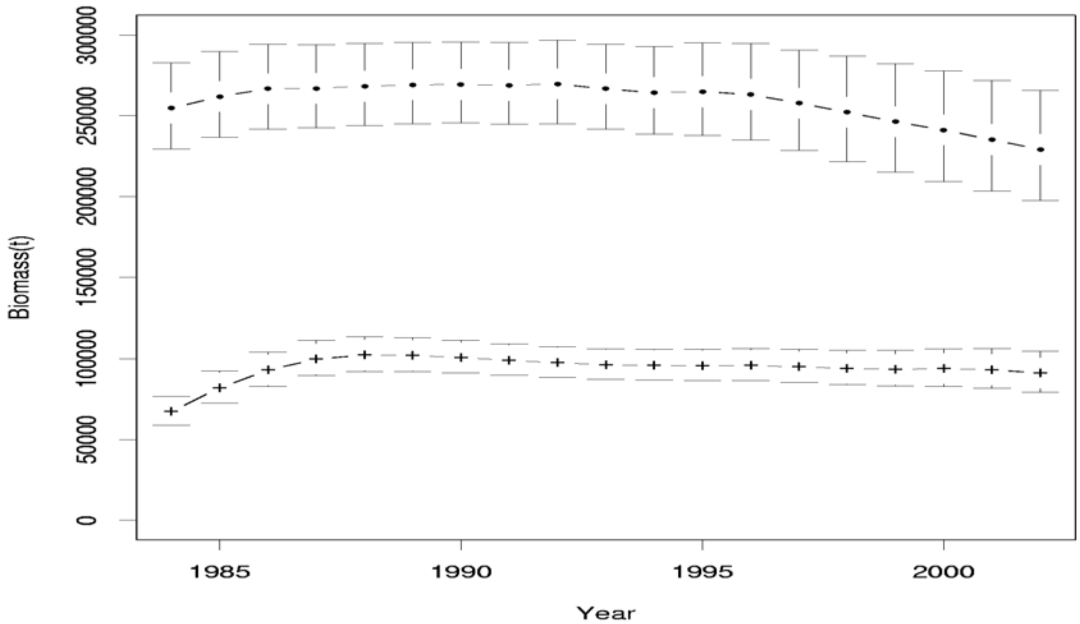


Figure 5.9. Age 3+ biomass (dotted line) and female spawning biomass (line with +) from 1984 to 2002.

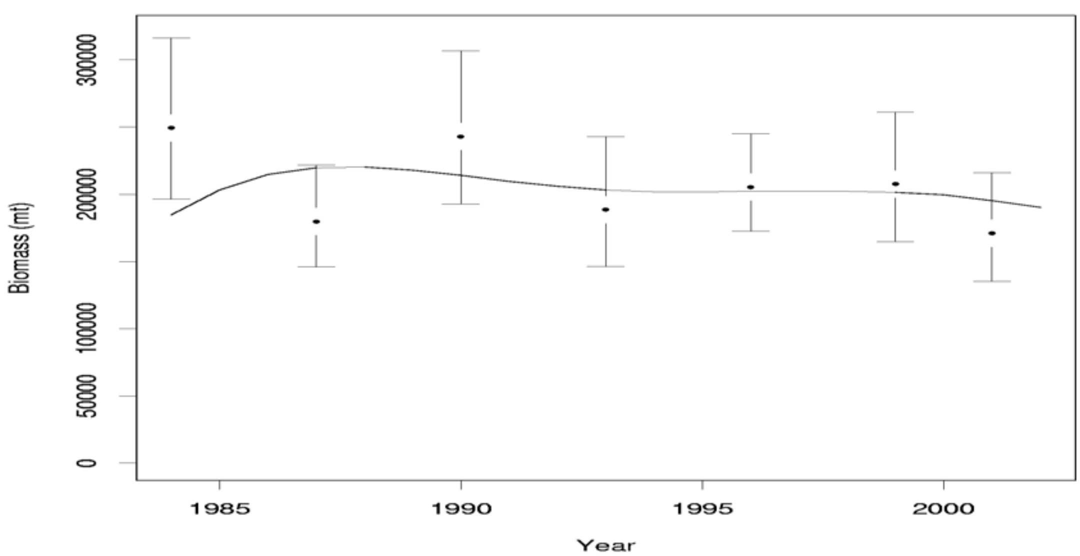


Figure 5.10. Fit to the survey biomass estimates.

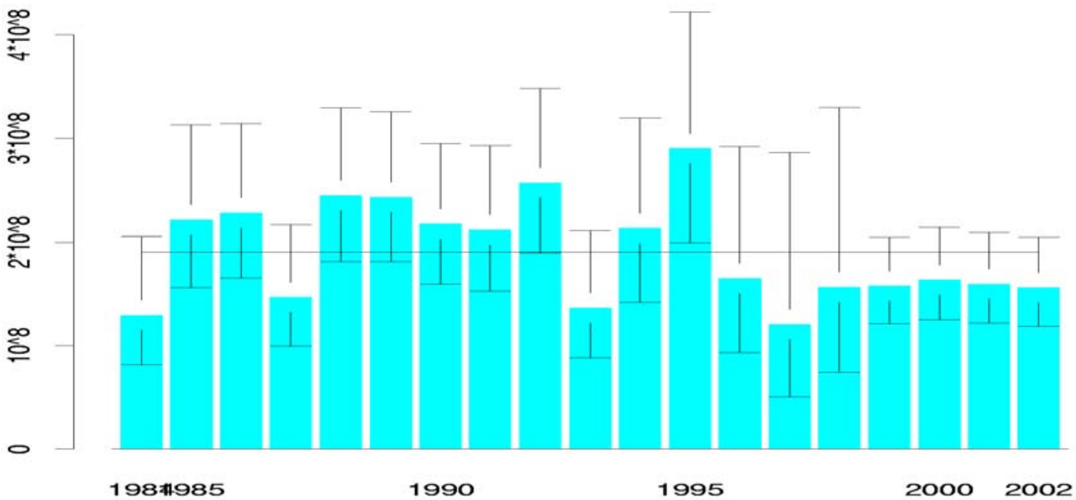


Figure 5.11. Age 3 recruitments from 1984 to 2002 with approximate 95% lognormal confidence intervals. Horizontal line is mean recruitment.

Appendix A.

Table A.1. Model equations describing the populations dynamics.

| | | | |
|---|---|---|---|
| $N_{t,1} = R_t = R_0 e^{\tau_t}$ | $\tau_t \sim N(0,\sigma_R^2)$ | | Recruitment |
| $C_{t,a} = \frac{F_{t,a}}{Z_{t,a}}(1-e^{-Z_{t,a}})N_{t,a}$ | | $1 \le t \le T$<br>$1 \le a \le A$ | Catch |
| $N_{t+1,a+1} = N_{t,a} e^{-Z_{t,a}}$ | | $1 < t \le T$<br>$1 \le a < A$ | Numbers at age |
| $FSB_t = \sum_{a=1}^{A} w_a \phi_a N_{t,a}$ | | | Female spawning biomass |
| $N_{t+1,A} = N_{t,A-1} e^{-Z_{t,A-1}} + N_{t,A} e^{-Z_{t,A}}$ | | $1 < t \le T$ | Numbers in “plus” group |
| $Z_{t,a} = F_{t,a} + M$ | | | Total Mortality |
| $C_t = \sum_{a=1}^{A} C_{t,a}$ | | | Total Catch in numbers |
| $p_{t,a} = C_{t,a} / C$ | | | proportion at age in the catch |
| $Y_t = \sum_{a=1}^{A} w_{t,a} C_{t,a}$ | | | Yield |
| $F_{t,a} = s_a E_t e^{\varepsilon_t}$ | $\varepsilon_t \sim N(0,\sigma_R^2)$ | | Fishing mortality |
| $S_a = \frac{1}{1+e^{(-b(Age-A_{50\%})}}$ | | | Selectivity- 2 parameter ascending logistic for fishery- separate for males and females |
| $S_a = \frac{1}{1+e^{(-b(Age-A_{50\%})}}$ | | | Selectivity- 2 parameter ascending logistic for survey- separate for males and females |
| $SB_t = Q\sum_{a=1}^{A} w_a s^s_{t,a} N_{t,a}$ | | | survey biomass, Q = 1. |

Table A.2. Likelihood components.

| | |
|---|---|
| $\sum_{t=1}^{T}\left[\log(C_{t,obs})-\log(C_{t,pred})\right]^2$ | Catch using a lognormal distribution. |
| $\sum_{t=1}^{T}\sum_{a=1}^{A} nsamp_t * p_{obs,t,a}\log(p_{pred,t,a})$ - offset | age and length compositions using a multinomial distribution. Nsamp is the observed sample size. Offset is a constant term based on the multinomial distribution. |
| offset = $\sum_{t=1}^{T}\sum_{a=1}^{A} nsamp_t * p_{obs,t,a}\log(p_{obs,t,a})$ | the offset constant is calculated from the observed proportions and the sample sizes. |
| $\sum_{t=1}^{ts}\left[\frac{\log\left[\frac{SB_{obs,t}}{SB_{pred,t}}\right]}{sqrt(2)*s.d.(\log(SB_{obs,t}))}\right]^2$ | survey biomass using a lognormal distribution, ts is the number of years of surveys. |
| $\sum_{t=1}^{T}(\tau_t)^2$<br>$\sum_{a=3}^{15}(diff(diff(s_a)))^2$ | Recruitment, where $\tau_t \sim N(0,\sigma_R^2)$<br>Smooth selectivities. The sum of the squared second differences. |

Table A.3. List of variables and their definitions used in the model.

| Variable | Definition |
|---|---|
| T | number of years in the model(t=1 is 1984 and t=T is 2002 |
| A | number of age classes (A =18, corresponding to ages 3(a=1) to 20+) |
| $w_a$ | mean body weight(kg) of fish in age group a. |
| $\phi_a$ | proportion mature at age a |
| $R_t$ | age 3(a=1) recruitment in year t |
| $R_0$ | geometric mean value of age 3 recruitment |
| $\tau_t$ | recruitment deviation in year t |
| $N_{t,a}$ | number of fish age a in year t |
| $C_{t,a}$ | catch number of age group a in year t |
| $p_{t,a}$ | proportion of the total catch in year t that is in age group a |
| $C_t$ | Total catch in year t |
| $Y_t$ | total yield(tons) in year t |
| $F_{t,a}$ | instantaneous fishing mortality rate for age group a in year t |
| M | Instantananeous natural mortality rate |
| $E_t$ | average fishing mortality in year t |
| $\varepsilon_t$ | deviations in fishing mortality rate in year t |
| $Z_{t,a}$ | Instantantaneous total mortality for age group a in year t |
| $s_a$ | selectivity for age group a |

Table A.4. Estimated parameters for the model. There were 66 total parameters estimated in the model.

| Parameter | Description |
|---|---|
| $\log(R_0)$ | log of the geometric mean value of age 3 recruitment |
| $\tau_t$ $1984 \le t \le 2002$, plus 18 parameters for the initial age composition equals 37. | Recruitment deviation in year t |
| $\log(f_0)$ | log of the geometric mean value of fishing mortality |
| $\varepsilon_t$ $1984 \le t \le 2002$, 19 parameters | deviations in fishing mortality rate in year t |
| Slope and age at 50% selected – 4 parameters | selectivity parameters for the fishery for males and females. |
| Slope and age at 50% selected – 4 parameters | selectivity parameters for the survey data, for males and females. |

Table A.5. Fixed parameters in the model.

| Parameter | Description |
| --- | --- |
| M = 0.2 | Natural mortality |
| Q = 1.0 | Survey catchability |
| $L_{inf}$ , $L_{age2}$ , k , cv of length at age 2 and age 20 for males and females | von Bertalanffy Growth parameters estimated from the 1984-1996 survey length and age data. |